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A linear programming application to grain merchandising

Roger Cyrus Knapp
Iowa State University

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A LINEAR PROGRAMMING APPLICATION
TO GRAIN MERCHANDISING

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Roger Cyrus Knapp

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INTRODUCTION

The grain trade is one of the largest and most important industries serving American agriculture. The percentage of feed grain[1] sold off farms increased from 25 percent (22.3 million tons) in 1939 (1, p. 55) to 48 percent (75 million tons) in 1966 (2, pp. 34-35). Total value of off-farm feed grain sales in 1966 exceeded 3.7 billion dollars (2, p. 35). Foreign exports of grain have made a significant contribution to the credit side of the United States' balance of payments ledger. The value of feed grain exports during the 1967 fiscal year alone exceeded 1.1 billion dollars (3, p. 5).

Grain marketing is a complex operation involving physical facilities for transporting, storing, merchandising, processing and pricing grain and grain products. Grain merchandising, which is the subject of this inquiry, may be further divided into the following three stages: 1) the accumulation of grain in country, subterminal and terminal elevators, 2) the assortment of quantities of like grades and quality into relatively homogeneous lots, and 3) the allocation of the homogeneous lots of grain to processors and exporters. This study attempts to analyze some of the problems involved in the merchandising of grain and proposes an analytical technique for use in the determination of optimal solutions.

Statement of the Problem

Major changes and developments have taken place in the grain marketing

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[1]The feed grains include corn, oats, barley and grain sorghum.

industry within the past decade. Several of these changes were either directly or indirectly the result of the influence of the federal government. Direct market activities by the federal government, which are administered by the Commodity Credit Corporation (CCC), includes the acquisition and disposal of grain. The CCC handles and stores vast quantities of grain in government-owned facilities and also provides for these services under contractual agreement with privately owned firms. This acquisition and storage of grain is the most obvious area of governmental influence on the industry.

Stocks of government-owned feed grains in the United States increased from 868 million bushels in the first quarter of 1956 to 2.06 billion bushels during the first quarter of 1962 (2, p. 53). This expansion of the federal grain storage program molded the structure for a rapidly expanding grain storage industry.

During the 1950's, the Commodity Credit Corporation provided several incentives to encourage private investment in commercial storage facilities. Occupancy guarantee agreements were first offered in 1949 to private grain firms which constructed new storage facilities or which made additions to existing storage facilities. Under this program the CCC agreed to underwrite the occupancy of these new facilities. To provide special incentives to cooperative associations, Congress, in 1949, amended the Farm Credit Act of 1933 to allow the Bank of Cooperatives to loan up to 80 percent of the cost of new storage facilities constructed by farmer-owned cooperative associations. Additional incentives were provided through loans administered by the Small Business Administration and through the accelerated amortization provision of the 1954 Internal Revenue Code. The Code allowed warehousemen to construct grain storage facilities and depreciate the facilities for tax purposes over a five year (60 month) period.

The costs of federal farm price stabilization programs rose rapidly during the 1950's. During the fiscal year 1958, such programs cost the American taxpayers approximately $2.7 billion. More recently, during the first six months of the 1968 fiscal year, the Commodity Credit Corporation incurred total expenses of $44,025,000 for the express purpose of handling and storing grain (4, p. 2). A federal expenditure of this magnitude is capable of having a dramatic effect on any section of the economy.

With these incentives, existing firms improved and expanded facilities while new firms entered the industry to capture the lucrative payments available from the storage of government-owned grains. During the period from 1948 to 1963, the number of establishments classified as terminal elevators by the United States Bureau of the Census increased from 391 to 633 (5, p. 8).

In 1956, a representative annual storage rate for receiving, storing and loading corn was 21 cents per bushel.[1] Table 2 contrasts the proportions of total grain income received by the industry from the three major grain activities: CCC storage, grain merchandising and private storage. Approximately one-fourth of the income of the smaller grain elevators in the study came from the CCC in the 1950-56 period, whereas over one-third of the grain income of the larger firms came from the CCC.

Since 1962, total grain stocks owned by the CCC have been reduced sharply. CCC stocks of government feed grains during the first quarter of

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[1]See Table 1.

Table 1. Rates, per bushel, for storing and handling CCC corn[a]

	Approximate yearly storage charges[b]		Handling charges for trucked grain (actual)	
Year	Storage	Conditioning, insurance and other	Receiving	Load out
		Cents		
1946	7	1½	2 3/4	1/2
1947	7	1½	2 3/4	1/2
1948	9	2		1/2
1949	9	2	2 3/4	1/2
1950	10	4[c]		1/2
1951	10	4[c]	2 3/4	1/2
1952				
Commingled	13	4[c]		1/2
Iden. pres.	12	4[c]		1/2
1953				
Commingled	13	4[c]		1/2
Iden. pres.	12	4[c]		1/2
1954				
Commingled	15 3/4	4[c]		1/2
Iden. pres.	14 1/2	4[c]		1/2
1955				
Commingled	15 3/4	4[c]		1/2
Iden. pres.	14 1/2	4[c]		1/2
1956				
Commingled	16 1/2[d]		3 3/4	3/4
Iden. pres.	14 1/2[d]		2 1/2	3/4
1957-1959		Rates unchanged from 1956		
1967-1969[e]	13.14[d]		4	1 1/2

[a]Source: (6, p. 3).

[b]Approximate rates for one year's storage.

[c]Includes receiving charges.

[d]Includes all storage, conditioning and insurance charges.

[e]Wood, Paul, Director, Agricultural Stabilization and Conservation Service, Des Moines, Iowa. Data for 1967-69. Private Communications. 1968.

Table 2. Average gross income from grain merchandising and the CCC as a percent of average gross income from all grain sources for cooperative elevators by size group[a]

Period	Grain merchandising				CCC				Private storage			
	Small	Med-small	Med-large	Large	Small	Med-small	Med-large	Large	Small	Med-small	Med-large	Large
1926-29	100	100	100	100								
1930-33	100	100	99	98								
1934-41	96	86	91	84	11[b]	26[b]	17[b]	23[b]				
1942-45	91	89	86	82	9	11	13	11			1	7
1945-49	91	88	92	86	6	7	4	4	3	5	4	10
1950-56	70	60	56	51	26	36	35	37	4	4	9	14

[a]Source: (7, p. 20).

[b]The 1934-41 period figures are averages of only the three years, 1939-1941.

1968 were 462.6 million bushels, the lowest level in 15 years (2, pp. 34-35). The profits of firms engaged in storing government grain, which had previously been averaging several cents per bushel, dropped in many cases to a fraction of a cent. The reduction in government grain storage income has forced many of these firms to be faced with a problem of survival. Several of these firms, in addition to storing government grain, are also grain merchandisers.[1] It is important that new methods be developed at

[1]Grain merchandisers act as intermediaries between country elevator operators and grain processors and exporters. The merchandisers earn a profit by having a favorable profit margin between the purchase and resale price of the grain and/or from blending the purchased grain to improve the grade.

all levels of the grain trade to assist managers in developing more efficient methods of merchandising grain if these firms are to remain in operation.

Frequently the factors which need to be taken into account in making operational decisions are so numerous and complex that they cannot all be considered simultaneously, even by the most capable manager. Management would be greatly assisted by a systematic method of organizing pertinent information so that it can quickly make sound operational decisions. The technique of linear programming has proven to be a useful tool for analyzing problems similar to those facing the grain merchandiser and insures an optimum solution consistent with the coefficients and restrictions used in the problem (8, pp. 21-53). It is imperative to realize that the results of such a technique can only be as realistic as the accuracy of the coefficients used.

Objective

The objective of this study was to develop an analytical approach to managerial problems using the technique of linear programming. Primary emphasis was placed on presenting an operational model and explaining how it was designed. An attempt was made to develop a workable model that could be modified or expanded to meet specific situations.

In this paper the model was utilized to assist management in determining the optimal routing for various shipments of grain and for the determination of the least cost grain blend. The optimal solutions to these problems aids management in maximizing merchandising profits. The model could also be used to compare the economic advantages of artificially drying grain as versus selling off-grade high moisture corn.[1] Another possible use of the model would be to analyze the grade standards and discount rates currently in effect in the industry. It would be of interest to the industry to know how accurately the present discount rates adequately reflect the market value of the grain.

The technique of linear programming can be used to assist management in determining the most profitable solution to several of the industry's problems by simultaneously considering all available information relative to the problem. The author believes that when increased efficiency is realized in the marketing system, the producer and consumer will ultimately be the chief benefactors.

Review of Related Studies

It is difficult to trace the history of linear programming because several independent lines of thought cumulated in its successful development. During World War II, a mathematical technique was developed to determine the optimal shipping routes for movement of allied war material to overseas destinations. In 1947, a mathematician, George B. Dantzig (9), capitalizing on war time experiences, perfected a method for planning activities for the United States Air Force. This new technique analyzed problems which were a linear function of a number of variables to be maximized (or minimized) when these variables were subjected to a number of restraints in the form of linear inequalities. This technique became known as linear programming.

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[1] Refers to a grade which carries a price discount.

The solution of the linear-programming problem for the Air Force stimulated two lines of development. The first which is the subject of this study was the application of the technique to managerial planning. The relationship between the goals and activities of the Air Force in Dantzig's model was found to be analogous to the input-output relationship of the economy. The second area of development was undertaken by T. C. Koopmans (10) who explored the implications of this new approach to general economic theory.

One of the first applications of linear programming to industry was for the purpose of blending aviation gasoline. Additional minimization studies were conducted simultaneously by the feed industry. Waugh (11) investigated the practicality of using the technique as a tool to determine the minimum cost ration subject to specific nutritive requirements. Additional least cost ration studies were undertaken by Fisher and Schruben (12).

Programming studies aimed at profit maximization were first conducted during the mid-1950's. It was generally assumed in these studies that a limited amount of resources was available, the transformation process was subject to constant returns to scale, and an unlimited market at a known price existed for the finished product. The main objective of these studies was to prepare new study procedures rather than to test the actual application of the method.

In 1957, Scott (13) used the technique to program an actual firm's operations. The objective was to find the optimal combination of activities which would maximize a firm's profits.

To the author's knowledge, no studies have been attempted which deal specifically with problems of the grain merchandiser. The present study focuses attention on the grain inventory on hand to determine which qualities of grain contribute significantly to the firm's profits. The critical factor is the marginal value product[1] of the particular quality of grain when used in a blend of several lots of grain to yield No. 2 corn. Using this method, the merchandiser is able to arrive at the value of a particular shipment of grain. In addition the model can assist management in determining the least cost grain blend to use in filling order contracts consistent with the specific requirements of that particular order.

Characteristics of the Grain Firm Programmed

The firm selected for the study was an Iowa regional grain marketing cooperative which limited its marketing services to its 310 member cooperatives. The firm has two elevator facilities located near Des Moines, Iowa with a combined capacity of 8.3 million bushels. Both elevators have complete facilities for storing and handling grain. In addition there are two barge loading sites on the Mississippi River, a terminal truck elevator in eastern Iowa and a terminal elevator located in East Chicago, Indiana.

The primary business activity of the firm is merchandising grain. Grain is purchased from member companies for resale to processors and exporters. The firm operates on approximately a .5 cent margin between purchase and resale price which is assumed to be just sufficient to cover the administrative cost of the transaction. Profits from the merchandising

[1]The marginal value product is defined as the addition to total revenue attributable to the addition of one additional unit of variable input, all remaining input factors held constant.

operations must be realized from either the appreciation in value of the grain inventory or from the process of grain blending.

Grain by nature is a heterogeneous product. The use of a grading system is a means of converting this heterogeneous product into more homogeneous lots. If measures of quality are continuous, the assignment of grade standards is an arbitrary process. One goal of designating grade standards is to attempt to achieve less quality variation within than between grades. It would be impossible for grain producers and traders to buy and sell intelligently without precise product descriptions. It has long been recognized that the establishment of quality standards is a necessary function of government if the grade specifications are to be standardized. In 1916, Congress passed the United States Grain Standards Act which established standards for six separate grades of grain.[1] These standards are based on the moisture content, test weight, percent foreign material, percent total damage and the percent heat damage of the grain. The grades are lowered for failure to meet the standards on any one of these five factors.

The process of grain blending dates back to the early years of the grain trading industry. It was obvious to grain dealers that it would be possible to blend several sub-standard grades which were discounted for different factors and arrive at a mix which would meet the grade standards required. The technique of blending became more popular after 1916 with the passage of the Grain Standards Act and the process is now widely practiced in the industry.

The firm in this study has adequate facilities for conditioning, blend-

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[1] See Table 3.

ing and storing grain. In addition to the blending operation grain is also purchased from country points for direct shipment to processors and exporters to fill open contracts. Of the 75 million bushels purchased in 1967, approximately 16 million bushels were routed to the elevator facilities located near Des Moines. This study deals specifically with the larger of these two elevator structures which has 252 separate grain holding bins. Of these bins, 228 have a capacity of 18,000 bushel each with the capacity of the 24 remaining bins limited to 9,000 bushel each. In addition, there are three grain storage tanks with capacities of approximately 500,000 bushel each. The facility was constructed with a heat detection system to measure increases in bin temperatures which is an indication of grain deterioration in the respective bins. Two continuous conveyor belts, running the length of the structure, move the grain to and from the bins. The elevator is equipped with a 2,000 bushel per hour continuous flow dryer and a large vibrating screen for the removal of foreign material from the grain. There are also facilities for loading and unloading box and hopper rail cars as well as trucks.

From the central office of the firm daily purchase bids are telephoned to country elevators. Once a bid is accepted, management must make the decision of where to move the grain. A large percentage of the grain which is purchased passes through the Des Moines rail yards where the grain is graded by federally licensed inspectors from the Des Moines Grain Exchange. Management now has the alternative of either shipping the grain directly to processors and exporters to fill open contracts or to route the grain through the elevator facilities in an attempt to gain a profit from blending.

When grain is routed to the elevator, it is segregated according to quality and stored in separate holding bins. To gain a blending profit, the grain for resale is combined with other qualities of grain and ideally the mixture will just meet the minimum grade specified in the contract. For grading purposes, a sample of grain must meet all the grade standards specifications. Unless stated in the buyer's contract, there is not any premium for exceeding these minimum standards. Basic contracts are usually written on the basis of No. 2 corn with all shipments which fail to meet this standard subject to a price discount. The discount scale currently used by this firm is presented in Table 4.

The firm's decision to use a shipment of grain for blending or to ship the shipment directly to a buyer is currently based on the total value of price discounts present per bushel. The value of price discounts per bushel is calculated by subtracting the discounted price of the grain from the base price of No. 2 corn. For example, using the discount scale presented in Table 4, the price discount for a bushel of 17 percent moisture corn would be 3 cents (17.0% - 15.5% x 2 cents/percent). In addition, grain which exceeds the minimum standards for No. 2 corn but can be purchased for the base price is also routed to the elevator. The objective of grain blending is to blend these various qualities of grain and arrive at a mix which will just meet the minimum standards set by law for No. 2 corn. All grain which meets these standards can be sold at the base price and is not subject to discounts.

Management is currently operating on the assumption that a shipment of grain must be discounted at least 2 cents per bushel to offset the cost of handling and blending the grain before the blending operation can show a

Table 3. Official USDA grade requirements for corn[a]

Grade	Min. test weight lbs./bu.	Maximum limits of			
		Moisture %	% Broken corn & foreign material	Damaged kernels	
				% Heat	% Total
1	56	14.0	2.0	0.1	3.0
2	54	15.5	3.0	0.2	5.0
3	52	17.5	4.0	0.5	7.0
4	49	20.0	5.0	1.0	10.0
5	46	23.0	7.0	3.0	15.0

[a]Source: (14, p. 3).

profit. The superintendent of the grain elevator then determines those lots of grain which will be used in a blend based on mental calculation and experience.

The process of grain blending is further complicated by the special characteristics of agricultural products, noteably their lack of uniformity. When unloading grain, foreign material and high moisture grain tend to move slower and exit in pockets. To compensate for the possible non-uniformity of the grain when blending, the superintendent insures that all blends are composed of grain from at least 17 lots. If necessary, the grain is screened to remove foreign material and dried to lower the moisture content in order to meet the requirements of the contract.

Management of the firm is confronted daily with the following decisions: which shipments of grain should be routed to the terminal elevator for use in the blending operation and which qualities of grain should be

Table 4. Off-grade discount scale

MOISTURE DISCOUNTS

2¢ per percent of moisture in excess of 15.5 percent

TEST WEIGHT DISCOUNTS

53.0 to 53.5 - 1¢	51.0 to 51.5 - 3¢
52.0 to 52.5 - 2¢	50.0 to 50.5 - 4¢

Market difference for all grain under 50#

DAMAGE DISCOUNTS

1/2¢ per percent of damage in excess of 5 percent
Market difference for all grain in excess of 15 percent

FOREIGN MATERIAL DISCOUNTS

3.1 to 4% - 1¢	4.1 to 5% - 2¢

2¢ discount for each additional 1% or
fraction thereof in excess of 5%

combined for the least cost blend to fill a contract. This study presents a method to evaluate the alternatives available to management.

Corn grades may be lowered for failure to meet established standards on any one of five factors. These are moisture, foreign material, test weight, total damage, and heat damage. Official standards for corn grading No. 1 through No. 5 are shown in Table 3. Corn not meeting the requirements for any of grades No. 1 through No. 5 is classified as sample grade. In addition, corn quality may be lowered to sample grade if the grain contains stones, is sour, musty or heating, or if it has an objectionable odor. The official standards define not only the grade requirements, but also the procedures and equipment to be used in sampling grain and in evaluating quality factors.

THE THEORETICAL MODEL

The model used in this analysis is based on the theory of the firm where the firm is defined as a profit maximizing, decision-making unit. The theory of the firm assumes that decision making within the firm is carried out by means of marginal analysis. Cohen and Cyert (15) define marginal analysis as the process of making a choice between alternatives by considering small changes in total satisfaction resulting from small changes in the combination of alternatives. In mathematical terminology, the marginal concept is the rate of change of an economic function with respect to the change in a continuous independent variable.

The theoretical firm, which is the basis of this model, is defined as operating in the short run (possessing a given stock of physical facilities). The firm will produce a given output at minimum cost or conversely, will maximize output for a given cost outlay. Thus, the firm will choose that combination of input factors which will allow it to produce a given level of output at a minimum cost. In order for the firm to maximize returns over expenditures, it must find the solution to three fundamental economic questions: 1) what is the optimal combination of outputs, 2) what is the optimal combination of inputs, and 3) what is the optimal level of production.

In the determination of the optimal mix of outputs which the firm should produce, consider the case of a firm using V units of input per unit of time to produce two outputs. Let P_1 and P_2 be the selling price of the two outputs, then the firm's total revenue function is

$$TR = P_1 q_1 + P_2 q_2. \qquad 1$$

Figure 1 is a graphical representation of Equation 1 for various values of q_1 and q_2. The straight lines labeled TR_i are isorevenue curves and represent the locus of all possible combinations of the outputs which result in the same total revenue where $TR_1 < TR_2 < TR_3 < TR_4$.

Figure 2 is a graphical representation of a set of contour lines called product transformation curves. Each product transformation curve is the locus of output combinations which can be obtained from a given level of inputs. Each of the curves labeled V_1, V_2, V_3 and V_4 represents a specific input rate. The contour V_j indicates all possible combinations of the two outputs which could be produced when V_j units of the input factors are used in production.

In Figure 3, the author has superimposed on the isorevenue curves from Figure 1 one of the product transformation curves from Figure 2. The point of tangency between the product transformation curve V_j and the isorevenue curve TR_i determines the combination of outputs which gives the firm the highest total revenue when V_j units of input are used in production. At the point of tangency, the slopes of the curves are equated, thus signifying the equality of the marginal rate of transformation of the outputs with the ratio of their prices. Thus, the first condition for profit maximization is that the rate of product transformation between every pair of outputs, holding all other outputs and inputs constant, must be numerically equal to the inverse ratio of their prices.

The second basic problem which must be solved for the maximization of the firm's profits is the determination of the optimal input mix. For a firm utilizing N inputs to produce one output, the short run profit function may be expressed by the following:

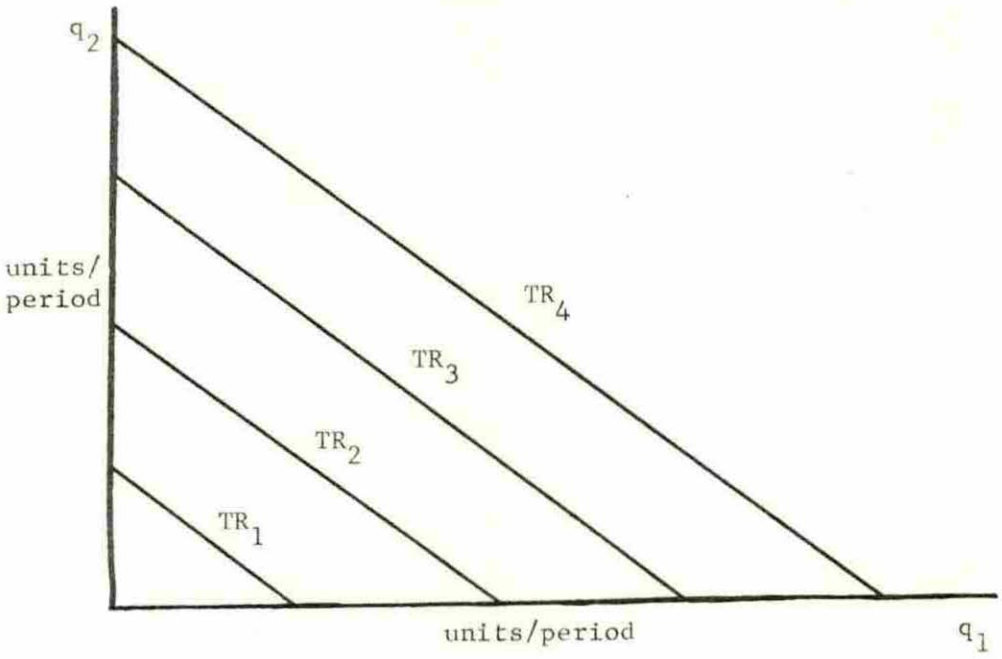

Figure 1. Isorevenue lines illustrating points of equal revenue

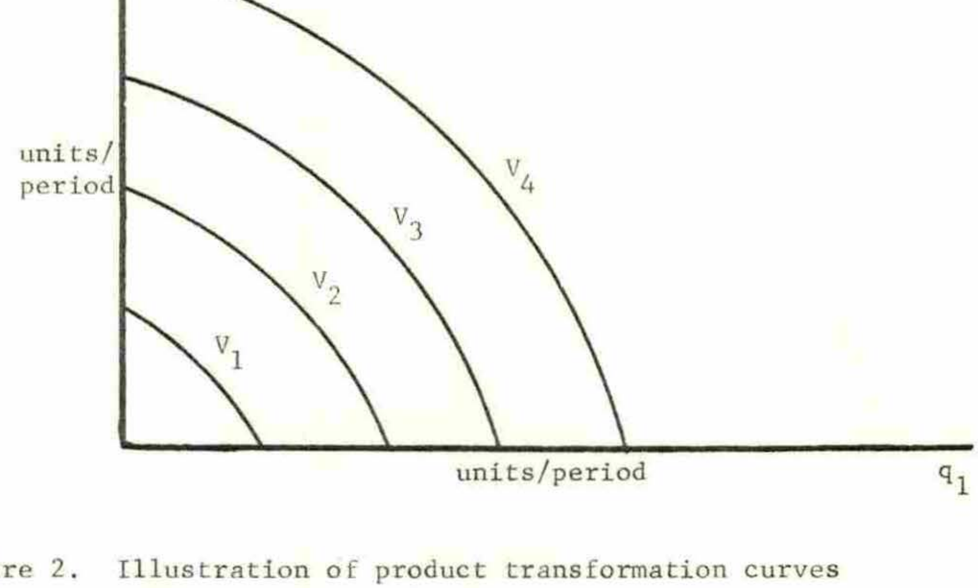

Figure 2. Illustration of product transformation curves

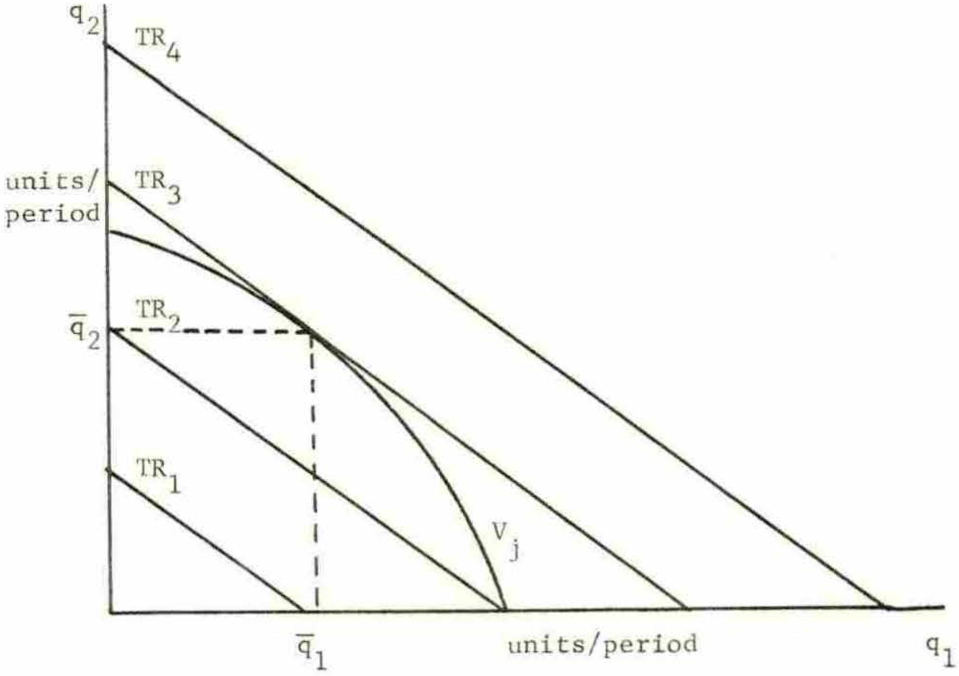

Figure 3. Illustration of the optimal combination of outputs

$$\pi = P_o Q - \sum_{i=1}^{N} P_i X_i - A. \qquad 2$$

Where P_o is a constant price at which quantity Q of output can be sold and P_i is the constant price at which input X_i can be purchased. Equation 2, by definition, is a short-run function; therefore the cost of fixed factors need not be shown explicitly but their influence is reflected in the function by the presence of the factor A. In the short run, fixed costs are defined as constant and therefore economic decisions are a function of only variable costs.

The production function is represented by:

$$Z = f(X_1, X_2, \ldots, X_i, \ldots, X_N) \qquad 3$$

where Q denotes the quantity of output and the X_i's are factors of production. The equation expresses the maximum amount of output that can be produced from any specified set of inputs, given the existing technology. A firm cannot maximize its profits unless it is operating on its production function.

The cost function can be represented by

$$C = P_1 X_1 + P_2 X_2 + \ldots P_i X_i + \ldots P_N X_N \qquad 4$$

where C represents the total variable costs, and the P_i represents the cost of each input factor. The firm attempting to maximize profits will maximize the production function subject to the cost restraint. A constrained maximization problem must be solved to determine the most profitable production decision.

To solve the constrained maximization problem, a Lagrangean function is formed to solve the problem using differential calculus. The function appears in the form:

$$Q = f(X_1, X_2, \ldots, X_N) - \lambda \left(\sum_{i=1}^{N} P_i X_i - C \right) \qquad 5$$

where λ is the Lagrangean multiplier (15, p. 122). Equation 5 is differentiated with respect to the X_i's and the results are equated to zero.

$$\frac{\partial Q}{\partial X_i} = \frac{\partial f}{\partial X_i} - \lambda P_i = 0$$

$$\frac{\partial Q}{\partial \lambda} = - \left(\sum_{i=1}^{N} P_i X_i - C \right) = 0 \qquad 6$$

$$i = (1, 2, \ldots, N)$$

The solution to Equation 6 maximizes the firm's production function subject to the cost restraint where $\frac{\partial f}{\partial X_i}$ is defined as the marginal physical product of X_i. The necessary condition for maximum output is expressed by:

$$\frac{MPP_1}{P_1} = \frac{MPP_2}{P_2} \cdots \frac{MPP_N}{P_N} . \qquad 7$$

The factors are employed in the amounts equating the ratios of marginal physical products to prices. The sufficient condition for a maximization is that $d^2 Q < 0$ when $d Q = 0$.

From Equation 7 it can also be shown that

$$\frac{P_1}{MPP_1} = \frac{P_2}{MPP_2} = \cdots \frac{P_N}{MPP_N} = MC \qquad 8$$

where MC equals marginal cost (16, pp. 169-173). It will be proven later that for maximum profits, a firm producing in pure competition must equate MC to P_o where P_o is the price of the firm's output, hence it follows:

$$\frac{P_1}{MPP_1} = \frac{P_2}{MPP_2} = \cdots \frac{P_N}{MPP_3} = P_o . \qquad 9$$

Equation 9 states that the value of the marginal product of each input is equal to the price paid for the input ($MPP_i \times P_o = P_i$). Accordingly, a necessary condition for maximization of profits is that all inputs be purchased in such quantities that the MVP's are equated to their factor prices.

The third problem which must be solved to maximize the firms profits is the determination of the optimal level of production. Again consider the case of a firm utilizing N inputs to produce one output. When Equation 2, the firm's short-run profit function, is maximized with respect to each X_i, the following is obtained:

$$\frac{\partial \pi}{\partial X_i} = P_o \frac{\partial Q}{\partial X_i} - P_i = 0 \qquad 10$$

$$i = (1, 2, \ldots, N)$$

The necessary condition for optimum output of a single product is given when

$$MPP_i = \frac{P_i}{P_o} . \qquad 11$$

The sufficient condition is given by $d^2 \pi < 0$ for any variable when $d \pi = 0$.

From Equation 11 it can be proven that

$$P_o = \frac{P_i}{MPP_i} .$$

It was shown in Equation 8 that a necessary condition for maximization of the firm's profits is expressed by

$$\frac{P_i}{MPP_i} = MC.$$

Thus, the optimal level of production of an output is the point where MC = P_o. Any output will be produced in such a manner that its selling price equals its marginal cost.

In analyzing the production decisions of a single firm it was assumed that factor and output market prices are constants. In the present study, the author has treated the solutions of two of these three basic economic problems as given: the determination of the optimal combination of outputs and the determination of the optimal level of production. Given these assumptions, the profit of the firm then becomes a function of the optimal combination of inputs which is the subject of this inquiry. The problem of profit maximization is now one of developing an analytical method whereby the firm can equate the marginal value product to input price.

THE LINEAR PROGRAMMING MODEL

The theory of the firm, as presented, shows the management of the firm as making decisions about one variable or at the most two variables at a time. In reality, the businessman must make decisions which are a function of dozens or hundreds of variables. Linear programming has been applied to a wide range of business problems to take into account such a multitude of variables. The economic meanings of linearity are constant returns to scale (MP = AP) and the prices of inputs and outputs are given and considered constant.

The central feature of linear programming is that it gives actual numerical solutions to optimization problems subject to a set of linear bounds or constraints. A linear programming problem has three components: an objective function, alternative processes or activities, and a set of constraints or restrictions. Any problem comprised of these three components can be expressed as a linear programming problem.

The objective function states the determinants of the quantity to be maximized or minimized. Profits or revenues are the objective function of a maximization problem; costs are the objective function of a minimization problem. The objective function may be expressed in physical, monetary, or other terms depending upon the problem being analyzed.

A process, also called an activity, is a particular method or technique of producing the enterprises to accomplish the objective. The programming procedure chooses among the alternative processes those most efficient in converting resources into the objective. Heady and Candler (8, p. 214) clarify what is meant by an activity in the following statement:

> "two production processes represent different activities if they (a) use different resources, (b) produce different products, (c) require different proportions of the same resources to produce the same product, or (d) use the same resources in the same ratios but produce products in different ratios."

Constraints or resource restrictions are the third component of a linear programming problem. Constraints are limitations or restrictions on the objective function. Given the three components of a programming problem it is possible to determine N feasible solutions. The objective of the programming procedure, however, is to determine the optimal solution which is the best of all possible feasible solutions as defined by the objective function.

In a profit maximization problem, the product mix for a firm is determined within the limits imposed by the constraints. In Figure 4 the geometric principles underlying the procedure of profit maximization are presented in a linear programming framework. The basic assumptions are that the firm produces two products X and Y subject to three linear constraints, A, B and C. The shaded area of Figure 4 is the zone of feasible production. Any combination of outputs X and Y within the zone is feasible but it is not possible to produce any combination of outputs outside the zone. The border of the production feasibility zone is the firm's production possibility curve. The relative prices of the outputs are accounted for by the slope of the isorevenue line represented by the dashed line in Figure 4. The intersection of the isorevenue line and the highest point on the firm's zone of feasible production, represented by point D, represents the solution to the optimal output mix problem. The results of the diagram show the quantities of each output to be produced for the maximization of profit

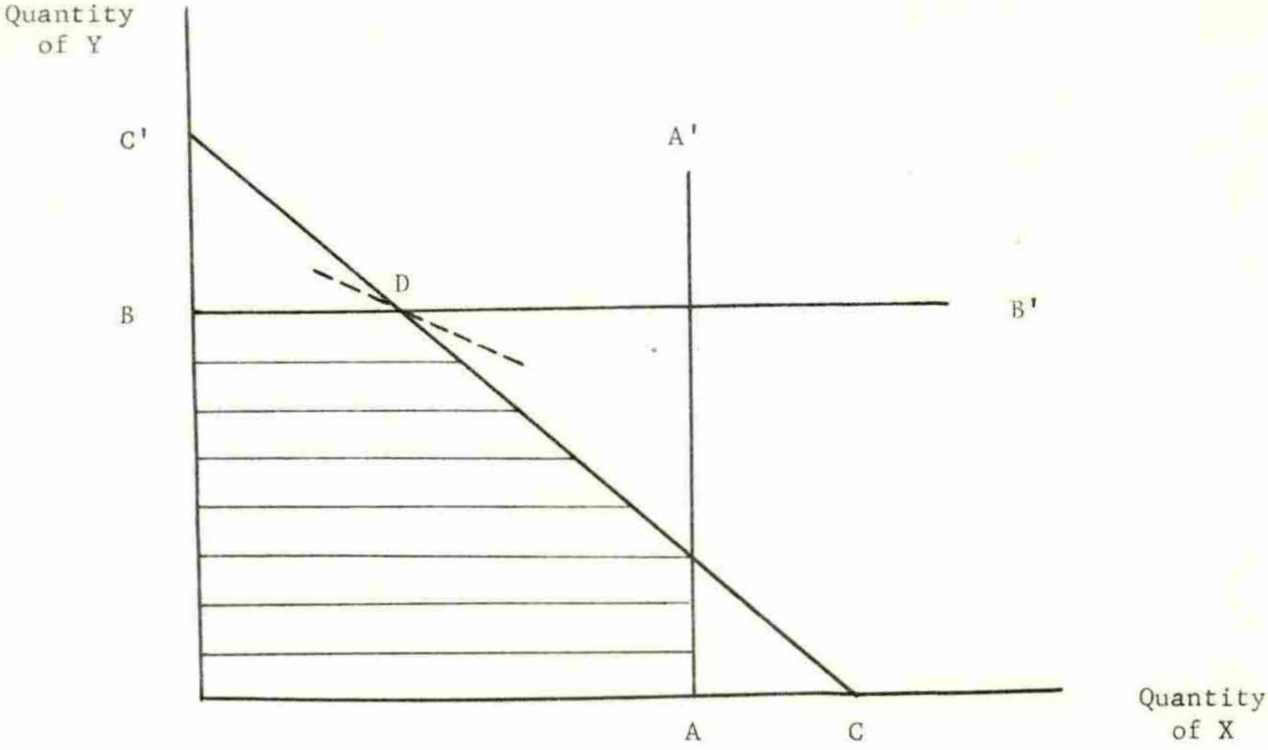

Figure 4. Illustration of the linear programming approach to profit maximization

subject to a set of three constraints.

The isorevenue line intersects a corner of the production possibility curve. The slope of the isorevenue line can vary greatly, caused by changes in the relative prices of outputs X and Y, and the optimal product mix will remain unchanged. If the isorevenue line would become tangent to a section of the border constraining the zone of possible production, there would not be a corner solution. Without a corner solution there cannot be a unique solution to the product mix problem. Any point between the adjacent left and right most corners of the point of tangency of the isorevenue line and the production possibility area are output combinations generating the same total revenue; hence a single unique solution would not exist.

The geometric principles behind the minimization problem are similar to those just presented except the objective is to minimize cost instead of maximizing profits.

In the full-mathematical form, not in the simplified version presented, linear programming is adaptable to a wide range of business problems. Businesses which are engaged in grain merchandising possess inventories of grain which are usually segregated according to quality factors into separate lots. When a selling order is prepared for shipment, several lots of grain are blended to meet the requirements specified for that particular order. The maximization of grain merchandising profits is represented by the following linear function,

$$\text{Max} \quad Z = P_j \, Q_j - \sum_{i=1}^{N} C_i \, X_{ij} \qquad 12$$

where Z = total net revenue from grain merchandising before fixed costs are subtracted for a given set of prices, costs and grade requirements. The additional terms are defined as follows:

P_j = selling price per bushel of the jth order,

Q_j = quantity in bushels of the jth order,

N = number of different lots of grain blended to fill the jth order,

C_i = purchase price or cost per bushel of the ith lot of grain, and

X_{ij} = quantity of the ith lot used to fill the jth order. It is the level at which the ith activity enters the final basis of the solution (17, pp. 79-80).

Equation 12 is maximized subject to the following linear restrictions:

$$\sum_{i=1}^{N} X_{ij} = Q_j \qquad 13$$

$$X_{ij} \leq Q_i \quad 14$$

$$X_{ij} \geq 0 \quad 15$$

$$\sum_{i=1}^{N} M_i X_{ij} \leq M_j Q_j \quad 16$$

$$\sum_{i=1}^{N} W_i X_{ij} \geq W_j Q_j \quad 17$$

$$\sum_{i=1}^{N} F_i X_{ij} \leq F_j Q_j \quad 18$$

$$\sum_{i=1}^{N} D_i X_{ij} \leq D_j Q_j \quad 19$$

$$\sum_{i=1}^{N} H_i X_{ij} \leq H_j Q_j \quad 20$$

The formulation is explained as follows:

Q = quantity in bushels of the ith lot or the jth order,

M = percent moisture in the ith lot or the jth order,

W = test weight per bushel of the ith lot or the jth order,

F = percent foreign material in the ith lot or the jth order,

D = percent damaged material in the ith lot or the jth order, and

H = percent of heat damaged material in the ith lot or the jth order.

Equation 13 states an equality condition. The number of bushels used to fill the jth order must exactly equal the number of bushels specified for the order. Equations 14 and 15 specify two additional conditions important from the standpoint of the mathematics of programming involved but obvious from a practical approach. Equation 14 expresses the condition that the number of bushels of the ith lot used to fill the jth order cannot exceed the amount in storage. Equation 15 states that the number of bushels of the ith lot used to fill the jth order cannot be less than zero. Equation 16 limits the moisture content. The percent moisture in the ith lot times the quantity of the ith type used cannot exceed the jth grade moisture restraint times the number of bushels in the jth order. Equation 17 expresses the condition that the test weight per bushel of the ith lot times the quantity of the ith type used to fill the jth grade order must be greater than or equal to the minimum test weight restriction for the jth grade times the number of bushels in the order. Equations 18, 19, and 20 are maximum restraints limiting the maximum amount of foreign material, damage and heat damage material respectively which may be included in an order.

Because fixed costs are not accounted for in the model, the final solution is a return to the fixed factors of production. The fixed costs are not included in the model because the optimal short-run economic plan is independent of the magnitude of fixed costs.

The above system of equations can be solved by relatively simple, though sometimes tedious, algebraic methods. For a detailed explanation of the steps involved see Dorfman, et al. (17, pp. 64-106). Without the aid of computers, the use of the linear programming technique would be greatly restricted.

The author maintains that the value of a particular quality of grain for blending purposes is a function of the entire grain inventory on hand as well as of the quality of the particular lot in question. When the objective function of a linear programming matrix is optimized with the aid of IBM's MPS/360 mathematical programming system routine, the marginal value product of each input factor can be determined. When the various qualities of grain programmed in the model are those currently held in inventory plus those shipments which the firm has the alternative of routing to the elevator, the programming routine gives management a practical analytical method of equating the marginal value product to the input price.

Determination of Coefficients and Prices

Where relevant data were available, the program coefficients were determined from the firm's records. In areas of the study where management's records were limiting, results from other research studies were relied upon. These studies were believed to be sufficiently accurate to be adaptable to this firm's specific situation. For example, it would be difficult to obtain accurate data from the firm's records for such items as the allocation of labor costs to the grain drying operation. The determination of costs was further complicated by the problem of allocating costs within a multi-product framework because the firm is engaged in two separate economic activities, the merchandising of grain and the storing of government-owned grain.

For the purposes of cost determination, the two central Iowa elevators operated by the firm were considered as one unit because the firm did not keep separate records for each facility. The cost and grain volume figures extracted from the firm's records covered the ten month period from September 1, 1967 through June 30, 1968. The average monthly operating expenses and the quantity of grain received per month were calculated from the above data.[1]

Operating costs per bushel

As discussed previously, management is assuming that grain handling and blending costs are 2 cents per bushel. In this section of the study the author attempted to arrive at a more accurate cost estimate.

Because the firm is engaged in two separate economic activities, a problem arises as to the proper allocation of elevator operating expenses between the merchandising and government grain storage activities. The two elevator facilities under consideration have a combined capacity of 8.3 million bushels. During the ten month period studied, an average of 45.8 percent (3.8 million bushels) of the total capacity was utilized for the storing of government-owned grain. The grain storage activity operates similar to the merchandising activity with the CCC grain, which is stored on a commingled basis, continually being received and outloaded. In addition, all grain in storage must be periodically turned and conditioned to maintain quality. Based on the assumption that the expenses for the two activities are similar, the author allocated expenses on a straight percentage basis. Of the $37,148.66 average monthly operating expense, 54.2 percent or $20,134.57 was charged to the merchandising activity. During this same time period, the firm received an average of 880,862 bushels of grain per month at the elevator facilities for merchandising purposes. Based on this data, an average operating cost of 2.29 cents per bushel was determined, excluding administrative costs and interest on fixed investment. It was further assumed that the .5 cent margin between purchase and

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[1]See Table 9, Appendix.

resale price was sufficient to cover the administrative expenses. Based on these figures, the returns from the blending operations must exceed 2.29 cents per bushel before the firm can either receive a return on invested capital or show a profit from merchandising grain.

Dryer operating costs

From the records of the firm it would be difficult to accurately estimate the costs of artificially drying grain. The drying cost coefficients used in this study are based on data collected by Harling (18). These cost coefficients were determined for a dryer with rated hourly capacity identical to that of the dryer operated by the firm in this study. The estimated annual volume of the two dryers were also approximately equal.

The cost figures in Harling's study were updated to reflect increases in operating expenses in the three years which have elapsed since the original data were collected. Harling's data, which were based on removing 10 percentage points of moisture, were further adjusted to yield coefficients based on the assumption that 5 points of moisture were removed. Both sets of coefficients with the additional assumptions are presented in Table 5.

The shrinkage losses resulting from artificial drying which are presented in Table 6 were determined by the use of the following mathematical formula:

$$\text{Shrinkage} = 1 - \frac{100 - \text{Initial Moisture \%}}{100 - \text{Final Moisture \%}} + .005$$

where the .005 compensates for the dry matter loss which accompanies the drying operation.

Table 5. Estimated cost of drying grain[a]

Rated dryer capacity - 2,000 bushels per hour
Estimated annual volume (bushels) 780,000

	10% points removed	5% points removed
	Cents/Bu.	Cents/Bu.
Fixed cost		
Depreciation	1.00	1.00
Insurance	.05	.05
Interest	.45	.45
Taxes	.21	.21
Total fixed cost	1.71	1.71
Variable cost		
Fuel	.60	.45
Electricity	.20	.14
Labor	.19	.13
Total variable cost	.99	.72
Total operating cost	2.70	2.43

[a]Source: (18, pp. 1-4).

The assumptions used in arriving at the above cost estimates are:

1. Fixed costs remain constant regardless of the points of moisture removed.

2. The variable cost for removing 5 points of moisture are greater than one half that of removing 10 points.

3. The dryer would be used for 20 hours per day for 30 days, 600 hours annually.

4. Fuel and power costs are constant; there is no allowance for temperature variations.

5. Repair and maintenance charges are not included. These costs would tend to increase with the age of the dryer, and would be offset in part by a reduction in interest cost resulting from lower loan balances as payments are made.

6. Administrative costs were not considered in the calculation of total cost.

Screening costs

In the screening operation the grain is removed from the bins, elevated to the top of the head house where it is passed over a 12/64 inch sieve and then the screened grain is returned to the storage bins. The screening sieve is perforated with round holes 0.1875 (12/64) inch in diameter which are 1/4 inch from center to center. The entire sieve, which is continuously vibrating, is powered by small air motors utilizing compressed air. As with the drying operation, it would be difficult to develop completely accurate cost coefficients pertaining to this operation. Any cost estimation determined would involve the arbitrary allocation of power and labor cost against the process. After consulting with management, the cost of screening was assumed to be .001 cents per percentage point of foreign material removed. In other words, 2 percentage points of foreign material could be removed from an 18,000 bushel capacity bin of grain with 5 percent foreign material for $36. As will be shown later in the study, the exact magnitude of the cost coefficients assumed is not of great importance because the computing routine used to solve the problem indicates the limits within which the activity is economically competitive.

Table 6. Percent shrinkage when grain is dried to selected moisture levels[a]

Initial moisture percent	Percent shrinkage when grain is dried to: 13.0%	13.5%	14.0%	14.5%	15.0%	15.5%	16.0%
13.5	1.07	0	0	0	0	0	0
14.0	1.65	1.08	0	0	0	0	0
14.5	2.22	1.66	1.08	0	0	0	0
15.0	2.80	2.23	1.66	1.09	0	0	0
15.5	3.37	2.81	2.24	1.67	1.09	0	0
16.0	3.95	3.39	2.83	2.25	1.68	1.09	0
16.5	4.52	3.97	3.41	2.84	2.26	1.68	1.10
17.0	5.10	4.55	3.99	3.42	2.85	2.28	1.70
17.5	5.67	5.12	4.57	4.01	3.44	2.87	2.29
18.0	6.25	5.70	5.15	4.59	4.03	3.46	2.88
18.5	6.82	6.28	5.73	5.18	4.62	4.05	3.48
19.0	7.40	6.86	6.31	5.76	5.21	4.64	4.08
19.5	7.97	7.44	6.90	6.35	5.79	5.23	4.67
20.0	8.55	8.01	7.48	6.93	6.38	5.83	5.27
20.5	9.12	8.59	8.06	7.52	6.97	6.42	5.86
21.0	9.70	9.17	8.64	8.10	7.56	7.01	6.46
21.5	10.27	9.75	9.22	8.69	8.15	7.60	7.05
22.0	10.84	10.33	9.80	9.27	8.74	8.19	7.65
22.5	11.42	10.90	10.38	9.86	9.32	8.78	8.24
23.0	11.99	11.48	10.97	10.44	9.91	9.38	8.84
23.5	12.57	12.06	11.55	11.03	10.50	9.97	9.43
24.0	13.14	12.64	12.13	11.61	11.09	10.56	10.03
24.5	13.72	13.22	12.71	12.20	11.68	11.15	10.62
25.0	14.29	13.79	13.29	12.78	12.26	11.74	11.22
25.5	14.87	14.37	13.87	13.37	12.85	12.33	11.81

[a]Shrinkage figures include actual moisture loss plus one-half percent for dry matter loss.

Table 6 (Continued)

Initial moisture percent	Percent shrinkage when grain is dried to: 13.0%	13.5%	14.0%	14.5%	15.0%	15.5%	16.0%
26.0	15.44	14.95	14.45	13.95	13.44	12.93	12.41
26.5	16.02	15.53	15.03	14.54	14.03	13.52	13.00
27.0	16.59	16.11	15.62	15.12	14.62	14.11	13.60
27.5	17.17	16.68	16.20	15.70	15.21	14.70	14.20
28.0	17.74	17.26	16.78	16.29	15.79	15.29	14.79
28.5	18.32	17.84	17.36	16.87	16.38	15.88	15.38
29.0	18.89	18.42	17.94	17.46	16.97	16.48	15.98
29.5	19.47	19.00	18.52	18.04	17.56	17.07	16.57
30.0	20.04	19.58	19.10	18.63	18.15	17.66	17.17
30.5	20.61	20.15	19.69	19.21	18.74	18.25	17.76

The screenings removed by the process consist mainly of grain dust and cracked corn. These screenings are in demand by livestock feeders in the local area who purchase the by-product on a per hundred-weight basis.

Additional factor input costs

All grain inputs were priced on a "to arrive" basis at the elevator site. A base price of $1.04 per bushel was assumed with all grades which failed to meet No. 2 corn standards being subjected to a series of price discounts. Each of the various qualities of grain under consideration was considered a separate activity. In addition, the model included a No. 2 corn merchandising activity with an objective function coefficient of $1.04, the base price per bushel of the grain. The base price for grain input factors and the selling activity were equated because the model did not include the administrative costs of the transaction which were assumed to be equal to the average merchandising margin.

Determination of Restraints

In the present study, the following restraints were incorporated into the model:

1. marketing limitations,
2. grade specifications,
3. capacity restraints,
4. factor variability limitations.

Purchase contracts for corn are written on the basis of No. 2 yellow corn. This marketing restriction therefore limits the alternative grade specifications which can be programmed into the model.

The grade specifications which must be met in all factors for No. 2 corn are:

1. test weight - 54 pounds per bushel minimum,
2. moisture - 15.5 percent maximum,
3. damage material - 5 percent maximum,
4. heat damage material - .2 percent maximum,
5. foreign material - 3 percent maximum.

All percents are determined on the basis of weight.

Due to the lack of information available on the amount of heat damage of the grain currently in storage, the fourth factor restriction was excluded from the model. The exclusion of this restriction should not seriously affect the results of the study since it was believed that only a small proportion of the grain had in excess of .2 percent heat damage.

The test weight of 54 pounds per bushel should not be confused with the legal weight per bushel as the two terms have different meanings. The test weight per bushel is defined as the weight of the volume of grain required to fill a Winchester bushel measure of 2150.42 cubic inches capacity. The legal weight per bushel is the number of pounds of grain required for a bushel without regard to volume and is the basis on which grain is bought and sold. The legal weight per bushel for corn has been fixed by federal law at 56 pounds (19, p. 12). The test weight figure is a determinant of quality and is used in assigning a grade designation to a lot of grain. The legal weight of a bushel of grain is always 56 pounds regardless of the test weight.

The restraint on legal weight became difficult to implement in the model when the grain inputs were defined in 56 pound units. Total weight was the most limiting restraint on the total number of bushels that could be blended. Because each grain input contributed an equal amount toward the total weight restriction, the marginal values of all input units were equated. For this reason, the activity units were defined in Winchester bushels and this problem was eliminated because the input units now varied in weight.

As previously defined, one Winchester bushel is equal to 2150.42 cubic inches of grain, the weight of which is equal to the test weight of the grain. To convert one Winchester bushel to one legal bushel, the test weight of the grain is divided by 56 pounds. The value of each input unit (Winchester bushel) was determined by the following formula:

$$\frac{\text{test weight}}{\text{56 pounds}} \text{ (base price - discounts)}$$

For a given grade of grain, the greater the test weight the greater is the value. Using Winchester bushels and the above formula compensates for the restraint that corn is purchased on a total weight basis.

The different qualities of grain in inventory were stored in separate storage bins and tanks. With the unit of activity defined as one Winchester bushel, the activity level of a particular quality of grain was restricted at a level equal to the bin or tank capacity where the grain was stored.

As previously explained, the firm stores government-owned grain in addition to its merchandising activities. Commercial elevators are not responsible for storing "identity preserved grain" as are farmers who seal on-farm grains as collateral against non-recourse loans. The elevator is permitted to store commingled grain which requires that there be in inventory the proper amounts and grades of grain to cover the outstanding warehouse receipts issued to the CCC. The grain in inventory may exceed the grade requirements, but it must not be of a lower quality than specified on the receipts. For the present study, 2 million legal bushels of No. 2 corn at 14.0 percent moisture were assumed necessary to cover the warehouse receipts.

A final restraint was imposed to off-set any variation in grain quality present within any lot. To average out quality variability, all grain for shipment must be obtained from 17 separate lots.

The Program Matrix

The theoretical and linear programming models have previously been presented. In Table 7 the general form of the linear programming matrix is presented as prepared for processing by the IBM MPS/360 mathematical programming routine. Activities P01 through PN represent the lots of grain

Table 7. General form of the linear programming matrix

	Activity							
Row	P01	...,	PN	PN+1	PN+2	PN+3	PN+4	PN+5
Weight	54.5		57.6	-.54	-5.23	-54.0		-54.0
% moisture	15.7		13.7	-.155	-5.0	-14.0		-15.5
% damage	3.0		4.4	-.05		-5.0		-5.0
% F M	2.6		2.0	-1.0		-3.0		-3.0
Screenings				-.54			100.	
C row	-1.002		-1.029	-.001	-.0243	1.002	1.37	1.002
Lower bound								
Upper bound	18,000		18,000					
Fixed bound						2,074,690		
$Z_i - C_i$	1.002		1.029	.001	.0243	-1.002	-1.37	-1.002

which the firm has available for merchandising. The unit of activity in each case is one Winchester bushel. The C row or objective function shows the discounted price or cost to the firm of each activity unit. Each of the activities was restrained at a level equal to the amount of the grain available. PN+1 and PN+2 are screening and drying activities respectively for conditioning the grain. The unit of activity for the screening activity is the removal of 1 percentage point of foreign material. The unit of activity for drying is the removal of 5 points of moisture. The other coefficients for these two activities reflect the additional average factor losses associated with the screening and drying operation. The foreign material screened from the grain is transferred to activity PN+4 where the screenings are assumed to be sold for $1.37 per hundredweight. PN+3, the activity covering the warehouse receipts, was forced into the program at a level of 2,074,690 (54/56 x 2,074,690 = 2,000,000). This manipulation accounts for the 2 million legal bushels of No. 2 corn at 14.0 percent moisture which is required to be held in inventory. Activity PN+5 is a No. 2 corn merchandising activity. The quantity of legal bushels available for merchandising was determined by multiplying the activity level of this activity by 54/56.

The Z-C row in Table 7 would not be included in the input matrix. The row is presented here for the purpose of showing its importance for the maximization of the objective function, $Z = C_1X_1 + C_2X_2 + \ldots, + C_NX_N$, of the mathematical model.

APPLICATION OF THE MODEL

At this point coefficients for the activities considered have been presented and assembled in a linear program matrix. This basic model will now be used to analyze two of the problems facing the grain merchandiser.

Optimal Grain Routing

Management of the firm makes the decision on whether to ship grain directly to buyers or to route the grain to the elevator for blending purposes based on the difference between the base price of No. 2 corn and the input cost of the shipment. Management, operating on the assumption that elevator operating costs are 2 cents per bushel, routes all shipments which are discounted 2 cents or more per bushel to the elevator for blending. Grain may be stored at the elevator in anticipation of a price increase, but this study further assumes that all warehouse grain was routed to the elevator to capture a blending profit.

The author maintains that the value of any grain shipment, when used in a grain blending operation, is dependent on the combined quality of the other grain in inventory. The potential value of the shipment depends on how well it will mix with the other grain in inventory to yield a product just sufficient to meet the minimum standards for No. 2 corn. For example, a carload of grain with 10 percent damaged material will have a higher potential value when all of the other grain in inventory average less than 5 percent damaged grain. The blending value of this shipment will decline as the total amount of damaged grain in inventory increases because the quantity of No. 2 corn which a blend of these grains will yield decreases.

It is the author's contention that the amount below the base price at which the grain can be purchased is not an accurate indicator of the grain's potential value. Rather, the quality of the grain in inventory is as important a determinant of value as the quality of the shipment itself.

The marginal value product (MVP) of the ith input is defined as the marginal physical product of the ith input multiplied by the price of output ($MVP_i = MPP_i \times P_o$).

If all grain inputs were programmed into the model at a zero price, the programming routine used for analysis would determine the MVP of each input factor. The base price of grain, which was used to determine both the prices of inputs and output, is continually changing, however, and the MVP fluctuates with this price change. When all grain inputs are programmed into the model at their input cost, the programming routine will generate the net marginal value product of each input factor. The net MVP is defined as the total MVP less input cost ($MVP_i - P_i$). The net MVP of each grain input is the value at the margin of that quality of grain for blending purposes (i.e., the change in the objective function with the addition of one additional unit of grain holding all other variables constant). The net MVP of a quantity of grain for a given blend is then constant and does not fluctuate with changes in the base price.

In this section the model will be used to determine the value of 189 separate lots of grain which the firm had previously routed to the elevator.[1] To determine the net MVP for each quality of grain, a maximization problem was solved using the matrix presented in Table 7. The objective

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[1]See Table 13, Appendix.

function coefficients were the discounted prices of the Winchester bushels priced on a "to arrive" basis. The activity covering the warehouse receipts entered the program at a level of 2,074,690 (2 million legal bushels), the level which the activity was forced into the program. The No. 2 corn selling activity entered the solution at a level of 2,854,923 Winchester bushels (2,752,716 legal bushels), indicating that this volume of grain was available for merchandising. The profit from the entire blending operation was $108,803 or an average of 2.29 cents per bushel. Two million bushels of the grain were not merchandised but were held in inventory. Allocating the blending profit to only the 2.75 million bushels available for sale gave an average return of 3.95 cents per bushel.

The screening and drying activities did not enter the final solution indicating that all grain could be blended to meet No. 2 corn standards without the need for screening foreign material or drying excess moisture. The shadow prices for these two activities which were -.76 cent and -10.6 cents respectively showed that profits would have decreased by these amounts per unit of activity (one Winchester bushel) if these two activities had been forced into the final solution. The values assigned to the objective function coefficients for these activities are not of prime importance to the results of the study because the programming results gave the limits within which these activities would be competitive. In this example, the C row coefficients for the screening and drying activities would have to have been greater than a positive .66 cent for the screening activity and 8.2 cents for the drying activity before these processes would have entered the program solution.

The net marginal value products of the various qualities of grain analyzed are presented in Table 11, Appendix. The results show the net MVP of a Winchester bushel of each grain input. Because the input costs of the Winchester bushels were determined on a weight basis, the results are interpreted also as the net MVP of a legal bushel of grain. These results indicate that 76 percent (144) of the lots possessed net MVP's in excess of price discounts. Of the remaining grain, 17 percent (32) of the lots had net MVP's equal to their discounts while for the remaining 7 percent (13) of the lots the discounts exceeded the net MVP. The value of discounts associated with a bushel of grain were determined by subtracting the input cost from the base price. From these results it can be concluded that the value of these price discounts associated with a bushel of grain are not equivalent to the potential value of the grain as currently assumed by management.

Of the 189 lots, 120 had net MVP's of less than 2.29 cents per bushel. Assuming operating costs per bushel to be 2.29 cents, as previously determined, the firm is reducing its profits by using these 120 lots for blending. The results further show that six of the lots possessed negative net MVP's. Even with operating costs assumed to be zero, using these lots for blending purposes would result in a monetary loss. From the Range Analysis[1] included in the programming routine, it was determined that the values of the net MVP's remained constant within the limits considered in the model.

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[1]The Range Analysis is used postoptimally to generate an analysis of the current solution. This analysis includes: (1) the effects of cost changes on optimal activity levels, (2) the cost of changing an activity from optimum level and the activity range for which this cost is valid, and (3) the value of changing the row activity level and the interval for which this value is valid.

In other words, for a given lot of grain, the MVP of the first bushel used for blending is the same as the MVP of the last bushel.

Based on these results, it was concluded that the marginal value product approach to grain quality evaluation is a sufficiently accurate analytical method to be of assistance in managerial decision making. When management is confronted with the decision of determining the destination of a carload of grain, the quality of that shipment can be programmed into the model. Once the net MVP of the grain is calculated, it can be compared with the firm's operating expenses. When the net MVP exceeds the cost of handling the grain, it would be beneficial to route the grain to the elevator for blending. If the cost of handling was greater than the net MVP, the firm could maximize profits or minimize its losses only by routing the shipment to a buyer. This approach also indicates to management the quality of grain that they should attempt to obtain from the country points and the premium that could afford to be paid, if necessary, to acquire the grain.

The Optimal Grain Blend

A second problem confronting managers of grain merchandising firms is how to maximize profits from a given inventory of grain. The objective of grain blending is to develop the least cost mix from the various qualities of grain available which will just meet the requirements specified in the buyer's contract. Unless stated in the contract, there is no economic advantage for the firm to exceed these requirements.

The model developed for this study will be used to determine the optimal blend. Each of the storage bins were again restrained at the level of maximum capacity. The three storage tanks were deleted from the program in order to comply with a subjective restraint of management. Management did not wish to use this grain for blending purposes because of the comparative difficulty encountered in moving the grain to and from the tanks. Because government grain is stored on a commingled basis, all grain in inventory was available for blending and the equality restraint covering the outstanding warehouse receipts was relaxed. The last manipulation was to force the No. 2 corn merchandising activity into the program at an activity level of 300,000. This equality restraint insures that the programmed mix will consist of grain from at least 17 separate lots (300,000/18,000). This restraint is intended to compensate for any quality variation present within bins. The model was then programmed to maximize returns over input costs. The activity levels of the lots which entered the solution are presented in Table 8. The results of the program gave the least cost blend which is just sufficient to meet the requirements for No. 2 corn. When these results were expressed in 56 pound units, it was determined that 289,241 legal bushels of No. 2 corn would be blended by combining this grain for shipment. With the assumption of a zero price margin between the price of inputs and the price of the output, the gross returns were $306,000 (300,000 Winchester bushels @ $1.002). The gross returns less the costs of inputs of $292,517 gave a profit from blending of $13,483 or a return of 4.66 cents per legal bushel blended.

Results

Previously it was shown that a necessary condition for maximum profits is that all inputs be purchased in such quantities that the MVP's are

Table 8. The optimal grain blend

Lot no.	Activity level	Price per Winchester bu.	Total cost	Quantity in legal bu.
108	18,000	.991	$ 17,838	17,838
132	3,696	.687	2,534	3,629
201	9,000	.948	8,532	8,982
222	9,000	.934	8,406	8,757
225	1,490	1.049	1,563	1,502
226	18,000	.950	17,100	18,000
240	18,000	1.058	19,044	18,324
304	18,000	1.087	19,566	18,810
321	18,000	1.039	18,702	18,144
325	18,000	1.049	18,882	18,144
327	18,000	1.049	18,882	18,144
329	9,000	1.059	9,531	9,162
424	18,000	.847	15,246	17,046
440	18,000	1.049	18,882	18,144
513	3,053	.982	2,998	2,946
525	18,000	1.049	18,882	18,144
531	18,000	1.059	19,062	18,324
534	18,000	1.047	18,846	18,486
536	18,000	1.058	19,044	18,324
537	18,000	1.054	18,972	18,324
Total			$292,517	289,241

equated to factor prices ($MVP_i = P_i$). In the present study, elevator operating costs were not included in the model. With these costs included, the necessary condition for maximum profits became $MVP_i = P_i + C$ where C is the total elevator operating costs on a per bushel basis. By transposing P_i to the left hand side of the equation, the following formula results: $MVP_i - P_i = C$. The left hand side of the equation was previously defined as the net MVP. This necessary condition for maximization of profits now becomes one of equating the net MVP to the average operating cost (net MVP = C). On this basis, the firm should use only those grain shipments for blending for which the net MVP exceeds the cost of handling the grain.

The results of the program showed that those qualities of grain with the lowest net MVP's were low in test weight. For the six lots of grain for which the input cost exceeded the MVP, the weight of a Winchester bushel of the grain was less than 54 pounds. Those qualities of grain with the highest net MVP's were those lots which were high in percentage of foreign material and could be purchased at a substantial discount. The foreign material could be added at low cost to the total blend. These results indicate that foreign material was not in excess supply. If an over supply of grain high in foreign material had been in inventory, the net MVP's of these lots would have been less.

The results of programming the entire inventory of grain showed that the average return per bushel blended was 2.29¢ which is exactly equal to the estimated operating costs. Because this figure does not include interest on fixed investment, it can be concluded that if all grain were blended for merchandising, the firm would be operating at a loss. All of the grain blended, however, was not merchandised because 2 million bushels were held in inventory to cover warehouse receipts.

The firm profits from the CCC storage activity in two ways. First, the firm receives a payment for storing and handling government-owned grain. Secondly, because the grain is stored on a commingled basis, the firm can profit by using this grain for blending. The warehouse receipts issued to the CCC were written for No.2 corn with 14 percent moisture. If the grain originally received from the CCC exceed the minimum requirements specified on the receipts, the firm can profit by using these excess factors. By blending all the grain, the firm is assured that the amount held in inventory does not exceed the minimum standards required. The firm is able to profit from this difference in quality between the government corn received and loaded out. By including the government-owned grain in the blend, the firm is able to sell more grain from a given inventory which meets No. 2 corn standards. This method allows the business to show a profit and continue to operate.

When Tables 8 and 11 are compared, it is noted that the lots of grain with the largest net MVP's are the quantities of grain used to blend the optimal grain mix. As these lots of grain are blended and shipped, the lots with lower net MVP's will be used for blending, and the cost of the mix can be expected to increase. The determination of the optimal routing of grain and the optimal grain blend then must be considered simultaneously. When a lot of grain is depleted, management must re-optimize the program solution to again determine the least cost blend which can be mixed from the inventory on hand. Management should then attempt to purchase those qualities of grain with the highest net MVP's to reduce the cost of future blends.

When a rise in temperature is detected in a bin, it is an indication that the grain is starting to deteriorate. If management wants to salvage the grain, the contents of the bin must be moved. By programming the quality of the grain into the model, the program will again determine the optimal blend subject to the added equality restraint that the total contents of that lot must be blended. In this example, the programming results indicate to management the most efficient manner of disposing of the grain.

In the determination of the optimal grain blend the drying activity did not enter the solution. The product could meet the requirements specified without the need for removing excess moisture. In the study, the author assumed that the only market available to the grain merchandiser was for No. 2 corn. Generally, this assumption can be described as realistic. There are, however, individual situations where the buyer may be willing to accept delivery of grain with a moisture content in excess of 15.5 percent. When confronted with this situation, the grain merchandiser has the alternative of either delivering No. 2 corn valued at the base price or an off-grade shipment sold at a discount.

When corn is bought and dried, fewer bushels will be sold and the total cost of the drying operation must be recovered on a reduced volume. The cost of shrinkage depends on the market value of the grain. The shrinkage cost for removing 4 percentage points of moisture is 7.6 cents per bushel when grain is valued at $1.50 per bushel but decreases to 5.1 cents with grain valued at $1.00 per bushel. While the value of the shrinkage varies directly with the value of corn, the discount rate for excess moisture remains relatively constant. It is, therefore, possible that for certain grain prices it would be advantageous for management to ship grain subject to a discount rather than suffer the shrinkage loss.

To assist management in selecting the most profitable alternative, the model was used to determine those combinations of grain prices and moisture levels for which artificial drying is profitable. Eight lots of grain at 16.5, 17.5, 18.5, 20.5, 22.5, 25.5, 28.0 and 30.0 percent moisture were added to the model. The basic structure of the model was modified to allow for the possibility of selling the high moisture grain subject to the price discounts which were presented in Table 4. The grain could also be dried to 15.5 percent moisture and sold at the base price. The per bushel costs of drying used were 2.70 cents for grain in the moisture range of 25.5 to 30.0 percent and 2.43 cents for grain from 16.5 to 25.5 percent moisture. The shrinkage losses were determined from Table 6. The last manipulation was to allow the base price of corn to vary from $.95 to $1.25 per bushel at 5 cent intervals. The shadow prices of the activities which failed to enter the final solution gave the profit or loss which would result from drying the grain.

The results of this section, presented in Table 12, Appendix, allows the grain merchandiser to determine the most profitable product for various prices, subject to the requirements of the buyer. The grade specifications of this product can then be programmed into the model and the optimal mix formulated.

Requirements for Application

The application of this linear programming technique requires access to computer facilities with mathematical programming capabilities. Key punch machines and operators must also be available because the data is recorded and stored on cards. The grain merchandising firm analyzed in this study does have such facilities available.

The largest expenditure of manpower will be required to initially construct the model and store the data on cards. The data deck for the program will require updating daily to reflect changes in the grain inventory. The time required to solve the problem varies with the size of the matrix. The matrix used in this study consisted of 194 columns and 6 rows. The total computer time required for solution averaged approximately 1.5 minutes. The costs of determining the optimal solution depends on the CPU and the Real time expended. The CPU time is the time required by the central processing unit to actively execute the program. This time was charged at the rate of \$375 per hour. The Real time is the total time the job was in the system and was charged at the rate of \$125 per hour. The actual costs of computing time varied from \$1.50 to \$3.00 per solution.

SUMMARY AND CONCLUSIONS

This chapter is included to allow the reader to comprehend quickly the problem, the model used for analysis and the results obtained and their implications for management.

The Problem

The grain merchandising industry operates with small profit margins. The margin between profits and losses can often be measured in fractions of a cent. The managers of grain merchandising firms are confronted daily with decisions dependent on more variables than can be comprehended simultaneously. Management would benefit and the efficiency of the grain trade would improve with the development of a practical, analytical method of analyzing the alternatives available. The mathematical procedure of linear programming has proven to be a useful technique for analyzing problems similar to those facing the grain merchandiser.

The Objective

The objective of the study was to develop an analytical technique which would prove to be of assistance in determining the optimal solutions to particular problems confronting grain merchandisers. An attempt was made to develop a workable model which could be adapted to meet individual situations.

The Model

The goal of the firm is assumed to be the maximization of profits which are defined in this study to be gross returns less total costs. Before the firm can maximize profits, the optimal solutions to three economic problems must be determined. These problems are: 1) what is the optimal combination of inputs, 2) what is the optimal combination of outputs, and 3) what is the optimal level of production? In the present study, the optimal combination of outputs and the optimal level of production are assumed to be given. The firm's profits are now a function of the optimal combination of inputs.

It was proven that a necessary condition for maximum profits is that all inputs be purchased in such quantities that the MVP's are equated to factor prices ($MVP_i = P_i$). When operating costs are included, this condition becomes equivalent to equating the net MVP to operating costs (net $MVP_i = C$).

A linear programming model was constructed which included all of the grain inputs available to the firm. The net MVP of each grain input was determined by solving the set of linear equations.

The assumptions included in the model are as follows:

1. The quality factors of all grain inputs are known exactly and without error.
2. When grain is purchased, management is limited to two alternatives. The grain can be either shipped directly to exporters and processors or the grain can be routed to the firm's elevator.
3. Management routed all warehouse grain to the elevator for the expressed purpose of using the grain to capture a blending profit.

4. Since a zero profit margin was assumed, the base price of inputs and outputs were equated.

Results and Implications

The linear programming model was used to analyze the grain merchandising activities of a large central Iowa cooperative association. To test the model, 189 lots of grain which the firm had previously routed to its central Iowa elevator were programmed into the model. The objective function of the linear model was maximized with the use of IBM's MPS/360 mathematical programming routine. The programming routine determined the optimal method of combining the 189 lots of grain in order to meet the minimum standards required for No. 2 corn. In addition, the programming routine determined the change in the objective function which would result from the addition of one additional bushel of each lot of grain holding all other variables constant. Because input costs were included in the model, these values are the net marginal value products of the various qualities of grain and must be equated to operating costs if profits are to be maximized.

The value of price discounts associated with a bushel of grain were determined by subtracting the input cost of the grain from the base price of No. 2 corn. Management currently determines the destination of a grain shipment from these values. Of the total 189 lots, 144 were found to have a marginal value in excess of these discounts. Of the remainder, 32 lots had a marginal value equal to the price discounts and 13 lots had a marginal value less than the value of discounts. Based on these results, the author concluded that the value of a particular grain shipment when used for blending is a function of the total inventory on hand. The value of the grain depends on how that shipment mixes with the present inventory to yield a product just sufficient to meet the minimum standards specified.

The results further showed that of the 189 lots of grain analyzed, 120 lots had a net MVP of less than 2.29 cents per bushel. With operating costs determined to be 2.29 cents per bushel as previously determined, the business is incurring a loss by using these lots for blending. Once these lots of grain were purchased, profits could only be maximized or losses minimized by shipping the grain to any available buyer at the prevailing price and not attempting to use the shipment in a blend. Based on these results it was concluded that the marginal value product approach to grain quality evaluation is a sufficiently accurate analytical method to be of assistance in managerial decision making.

Additional conclusions can be implied from the results of the program. When the warehouse receipts issued to the federal government are written on the basis of No. 2 corn, the grain in storage is not required to exceed these standards. By blending the entire inventory, the firm is assured that the minimum standards required by the receipts are met but not exceeded. When the quality of the CCC corn originally received exceed the minimum standards, the firm is able to profit by blending this grain.

The second application of the model was to determine the optimal blend from a given inventory of grain. Optimal is defined as the least cost blend which will just meet the requirements specified for No. 2 corn. The program solution gave the lot designations and the quantity of grain from each lot to be mixed to obtain the blend. The profit from blending was determined to be 4.66 cents per bushel. As expected, the quantities of grain with the highest net MVP's were the quantities which made up the least cost blend.

In summary, the linear programming model developed for the study gives grain merchandisers a practical analytical technique for evaluating the alternatives available. By continuously selecting the most profitable alternatives available, grain merchandising firms can continue to operate successfully with narrow profit margins.

Limitations of the Study

The study explicitly assumes that the grade factors of the grain are known without error. The grading, even though accomplished by licensed inspectors, is subject to human error. Furthermore, as the grain is transported, the percentage of foreign material can be expected to increase. The amount of damaged grain will also increase over time with improper storage. These sources of error must be allowed for before the model is applied to actual marketing problems.

The grade factors programmed into the model were expressed as a percent of total weight. Another possible source of error arises because of the variation in weight of the input units. In this study, it was believed that any discrepancies present were averaged out. This possible source of error could be eliminated in future studies by converting the percentage figures to pounds of actual material.

It was further assumed that when grain was purchased, management was confronted with two alternatives. Either the grain could be shipped directly to buyers or it could be routed to the elevator for blending. The actual movement of grain is restricted by the existing transportation facilities. Rail movement of grain in Iowa traditionally moves from west to east toward the Mississippi River and Chicago. For grain located at points east of Des Moines to be routed to the firm's elevator would require a back-haul because once the grain is blended and shipped it will again move in an eastward direction. This limitation of the study does not seriously affect the conclusions, however, because the optimal decisions can still be made within the existing transportation network.

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APPENDIX

Table 9. Average monthly operating expenses for the firm's elevator facilities[a]

Elevator supplies	$ 508.98
Fumigant	231.40
Power and gas	3,073.84
Insurance	1,158.18
Repairs and maintenance	1,390.01
Salaries	13,007.40
Retirement benefits	557.53
Taxes	10,552.90
Telephone	45.27
Truck expense	82.26
Locomotive fuel	60.86
Miscellaneous expense	282.95
Other	2,788.21
Depreciation[b]	3,408.87
Total	$37,148.66

[a]Calculated for the period September 1, 1967 through June 30, 1968.

[b]See Table 10, Appendix.

Table 10. Average monthly depreciation schedule for the firm's elevator facilities[a]

Depreciation	
Elevator A	
Head House	420.52
S. Annex A	112.21
S. Annex B	140.00
S. Annex C	748.31
Scale crib office	10.83
Walfare building	32.63
Head house machinery and equipment	2.34
Annex machinery and equipment	1.13
Fumigating system	4.49
	1,472.46
Elevator B	
Furniture and fixtures	18.05
Head house	7.32
Building A & B	.63
Building C & D	2.41
Butler building	2.87
Steel tanks	8.90
New shop	33.79
Head house machinery and equipment	184.49
Sec. A & B	391.25
Sec. C & D	49.73
Old shop	15.52
Other	964.39
South tank	13.20
North tank	205.07
Autos and trucks	38.79
	1,936.41
Total depreciation per month	3,408.87

[a]Calculated for the period September 1, 1967 through June 30, 1968.

Table 11. Evaluation of net MVP's and price discounts

Bin	Discount[a] (cents per bushel)	Net MVP
102	1	1.963
103	0	1.926
107	0	1.926
108	4	6.889
109	0	1.926
110	1	1.963
111	0	1.926
113	0	2.889
114	0	.963
115	0	.963
116	0	.963
117	7.5	7.500
118	0	.963
119	1	-.926
121	0	1.926
122	0	1.926
123	0	3.852
124	0	1.926
125	0	.963
126	4	.148
127	0	2.889
128	0	0
129	1	1.000
130	1	.037
132	34	35.926
133	0	3.852
134	0	.963
135	0	4.815
136	0	0
137	0	0
140	0	4.815
201	9	12.659
202	0	1.926
203	0	1.926
204	0	2.889
205	0	0
206	0	.963
207	0	1.926
208	0	.963
209	0	1.926

[a]The price discounts were determined by subtracting the input price of the grain from the base price of No. 2 corn.

Table 11 (Continued)

Bin	Discount[a]	Net MVP
	(cents per bushel)	
211	1	1.000
212	0	2.889
213	1	1.963
214	0	.963
215	1	2.926
216	0	.963
218	0	1.926
219	0	.963
222	8	8.963
223	1	-.926
224	0	3.852
225	0	4.815
226	9	12.852
227	0	1.926
228	0	0
229	4	7.852
230	0	2.889
231	0	.963
232	0	4.815
233	0	1.926
234	9	7.074
235	0	3.852
238	1	1.000
239	1	1.963
240	0	5.778
241	0	3.852
242	0	.963
243	0	1.926
301	1.5	3.426
302	0	.963
303	0	1.926
304	0	8.667
306	0	0
307	4	6.889
308	2	3.926
309	2	2.963
310	1	1.963
312	0	0
313	0	0
314	7.5	12.315
316	0	0
317	1	1.963
318	0	.963
319	0	0

Table 11 (Continued)

Bin	Discount[a]	Net MVP
	(cents per bushel)	
320	0	1.926
321	1	5.815
322	0	.963
323	0	0
324	0	0
325	0	4.815
326	1	1.000
327	0	4.815
328	0	3.852
329	0	5.778
330	3	-.852
331	0	4.815
332	0	4.815
333	7	6.037
334	0	.963
335	0	0
337	0	0
339	0	.963
340	1	-.926
401	40	43.852
403	0	1.926
404	0	1.926
405	0	0
406	1	-.926
407	0	0
408	0	1.926
411	43.5	46.196
416	0	1.926
417	0	.963
418	0	.963
423	0	.963
424	14.5	12.574
425	0	1.926
426	0	3.852
427	0	0
428	0	2.889
429	0	0
430	0	2.889
431	0	3.852
432	1	2.926
433	2	3.926
435	0	4.815
436	0	.963
437	0	0

Table 11 (Continued)

Bin	Discount[a] (cents per bushel)	Net MVP
438	0	3.852
439	0	0
440	0	4.815
441	0	4.815
442	0	2.889
443	0	4.815
501	0	.963
502	0	1.926
503	0	1.926
504	0	1.926
505	0	1.926
506	0	1.926
507	2	3.926
508	0	1.926
509	0	1.926
510	0	1.926
511	0	1.926
512	0	1.926
513	4	5.926
514	0	.963
516	0	1.926
517	0	0
522	0	2.889
523	0	2.889
524	1	-.926
525	0	4.815
526	0	1.926
527	0	0
528	0	0
530	0	.963
531	0	5.778
532	0	1.926
533	0	.963
534	2	8.741
536	0	5.778
536	1.5	6.278
538	0	4.815
539	0	1.926
540	0	4.815
602	0	0
604	0	1.926
605	0	0
606	0	1.926
607	0	1.926

Table 11 (Continued)

Bin	Discount[a]	Net MVP
	(cents per bushel)	
609	0	.963
613	0	3.081
615	0	0
616	0	.963
617	2	1.037
618	0	.963
619	0	0
621	0	.963
623	4	7.852
625	0	.963
627	0	0
629	0	.963
643	0	.963
644	3.5	1.574
tank X	1	5.815
tank Y	0	5.778
tank Z	0	6.933

Table 12. Profit or loss from drying corn to 15.5 percent moisture[a]

Original moisture content (percent)	Market value of No. 2 corn .95	1.00	1.05	1.10	1.15	1.20	1.25
30	9.517	8.633	7.750	6.867	5.983	5.100	4.217
28	7.769	7.004	6.239	5.474	4.709	3.944	3.180
25.5	5.566	4.948	4.331	3.713	3.095	2.478	1.860
22.5	3.231	2.792	2.353	1.914	1.476	1.037	.598
20.5	1.465	1.144	.823	.501	.180	-.147	-.462
18.5	.283	-.484	-.689	-.891	-1.094	-1.298	-1.499
17.5	-1.157	-1.300	-1.444	-1.587	-1.731	-1.874	-2.018
16.5	-2.031	-2.115	-2.199	-2.284	-2.368	-2.452	-2.536

[a]The drying costs assumed for removing 5 and 10 percentage points of moisture were 2.43 and 2.70 cents respectively.

Table 13. Inventory of grain on hand[a]

Bin number	102	103	107	108	109	110	111	113	114
Test weight	54.5	55.0	55.0	55.5	55.0	54.5	55.0	55.5	54.5
% moisture	15.7	15.0	15.0	14.5	15.5	14.7	15.5	15.5	15.5
% damage	3.0	1.4	1.0	3.0	3.0	3.3	3.0	4.0	2.5
% foreign material	2.6	2.9	1.7	5.9	2.4	3.4	2.5	2.9	2.7
Discounted price/ legal bushel	1.03	1.04	1.04	1.00	1.04	1.03	1.04	1.04	1.04
Input price	1.002	1.021	1.021	.991	1.021	1.002	1.021	1.03	1.012
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

[a]Includes all grain held in inventory by the firm on July 19, 1968.

Table 13 (Continued)

Bin number	115	116	117	119	121	122	123	124	125
Test weight	54.5	54.5	54.5	53.0	55.0	55.0	56.0	55.0	54.5
% moisture	15.3	15.3	16.0	15.2	15.3	14.0	10.7	14.5	14.2
% damage	2.9	3.0	18.0	2.2	3.4	4.0	2.0	2.0	3.5
% foreign material	2.7	2.4	2.8	2.9	2.9	1.0	3.0	2.5	2.8
Discounted price/ legal bushel	1.04	1.04	.965	1.03	1.04	1.04	1.04	1.04	1.04
Input price	1.012	1.012	.939	.974	1.021	1.021	1.040	1.021	1.012
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	126	217	218	129	130	132	133	134	135
Test weight	52.0	55.5	54.0	54.0	53.5	55.0	56.0	54.5	56.5
% moisture	13.5	14.5	15.0	15.0	14.3	12.3	13.9	13.4	13.8
% damage	4.2	4.0	3.0	3.5	2.2	60.8	3.0	2.5	3.5
% foreign material	4.3	1.6	3.0	3.5	1.9	6.9	1.3	2.9	2.3
Discounted price/ legal bushel	1.00	1.04	1.04	1.03	1.03	.70	1.04	1.04	1.04
Input price	.929	1.021	1.003	.993	.984	.687	1.040	1.012	1.049
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	136	137	140	201	202	203	204	205	206
Test weight	54.0	54.0	56.5	55.9	55.0	55.0	55.5	54.0	54.5
% moisture	15.5	14.8	13.7	14.5	15.5	14.7	15.5	15.3	15.5
% damage	4.0	2.0	4.7	14.9	2.0	2.6	4.0	3.5	2.7
% foreign material	2.9	2.5	2.0	5.4	1.8	2.9	2.3	2.9	2.6
Discounted price/ legal bushel	1.04	1.04	1.04	.95	1.04	1.04	1.04	1.04	1.04
Input price	1.003	1.003	1.049	.948	1.021	1.021	1.030	1.003	1.012
Bin capacity (bushels)	18,000	18,000	18,000	9,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	207	208	209	211	212	213	214	215	216
Test weight	55.0	54.5	55.0	54.0	55.5	54.5	54.5	55.0	54.5
% moisture	15.5	15.0	15.5	15.0	15.2	15.9	15.5	15.0	15.0
% damage	3.0	3.0	2.8	3.0	4.5	4.0	2.5	4.0	5.0
% foreign material	2.5	1.5	2.5	4.0	3.0	2.5	2.8	3.5	2.5
Discounted price/ legal bushel	1.04	1.04	1.04	1.03	1.04	1.03	1.04	1.03	1.04
Input price	1.030	1.012	1.021	.993	1.030	1.002	1.012	1.011	1.012
Bin capacity (bushels)	18,000	18,000	18,000	9,000	9,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	218	219	222	223	224	225	226	227	228
Test weight	55.0	54.5	54.5	53.0	56.0	56.5	56.0	55.0	54.0
% moisture	15.5	15.0	15.0	14.8	13.8	13.3	14.0	15.0	14.8
% damage	3.0	1.0	1.9	3.0	3.8	2.9	11.0	3.5	3.2
% foreign material	2.4	2.3	7.5	2.3	2.5	1.6	7.0	2.9	2.6
Discounted price/ legal bushel	1.04	1.04	.96	1.03	1.04	1.04	.95	1.04	1.04
Input price	1.030	1.012	.934	.974	1.040	1.049	.950	1.021	1.003
Bin capacity (bushels)	18,000	18,000	9,000	9,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	229	230	231	232	233	234	235	238	239
Test weight	56.0	55.5	54.5	56.5	55.5	53.0	56.0	54.0	54.5
% moisture	14.3	15.0	15.5	13.5	12.5	13.5	13.5	14.0	13.8
% damage	8.2	3.0	2.6	4.8	3.5	5.0	2.8	4.4	3.5
% foreign material	4.8	2.8	1.8	2.8	1.5	8.0	1.6	3.3	3.4
Discounted price/ legal bushel	1.00	1.04	1.04	1.04	1.04	.95	1.04	1.03	1.03
Input price	1.000	1.021	1.012	1.049	1.030	.899	1.040	.993	1.002
Bin capacity (bushels)	18,000	18,000	18,000	18,000	9,000	9,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	240	241	242	243	301	302	303	304	306
Test weight	57.0[1]	56.0[1]	54.5[2]	55.0[2]	55.0[2]	54.5[2]	55.0[2]	58.5[1]	54.0[2]
% moisture	14.0[1]	14.0[1]	14.3[2]	14.5[2]	15.5[2]	15.5[2]	15.3[2]	15.0[2]	15.2[2]
% damage	2.9[1]	2.0[1]	3.0[1]	2.5[1]	7.0[3]	3.0[1]	2.0[1]	2.7[1]	4.0[2]
% foreign material	1.6[1]	1.3[1]	3.0[2]	2.8[2]	2.5[2]	2.8[2]	2.7[2]	2.7[2]	2.9[2]
Discounted price/ legal bushel	1.04	1.04	1.04	1.04	1.035	1.04	1.04	1.04	1.04
Input price	1.058	1.040	1.012	1.030	1.026	1.012	1.030	1.087	1.003
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	307	308	309	310	312	313	314	316	317
Test weight	55.5	55.5	54.5	54.5	54.0	54.0	56.5	54.0	54.5
% moisture	14.5	15.3	15.3	15.8	15.5	14.0	14.3	15.0	15.7
% damage	12.5	3.0	3.0	4.5	3.3	3.9	17.3	2.5	3.0
% foreign material	2.4	4.4	5.0	2.6	2.9	2.3	2.5	2.9	2.7
Discounted price/ legal bushel	1.00	1.02	1.02	1.03	1.04	1.04	.965	1.04	1.03
Input price	.991	1.011	.992	1.002	1.003	1.003	.974	1.003	1.002
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	318	319	320	321	322	323	324	325	326	327
Test weight	54.5	54.0	55.0	56.5	54.5	54.0	54.0	56.5	54.0	56.5
% moisture	15.5	14.7	15.3	13.8	14.5	15.2	15.0	13.9	15.0	13.5
% damage	3.8	1.5	3.0	2.0	4.0	4.0	1.2	2.5	4.7	3.0
% foreign material	2.3	1.5	2.6	3.9	2.0	2.8	1.6	1.5	3.9	1.4
Discounted price/ legal bushel	1.04	1.04	1.04	1.03	1.04	1.04	1.04	1.04	1.03	1.04
Input price	1.012	1.003	1.021	1.039	1.012	1.003	1.003	1.049	.993	1.049
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	328	329	330	331	332	333	334	335	337
Test weight	56.0	57.0	52.0	56.5	56.5	53.5	54.5	54.0	54.0
% moisture	14.3	13.9	15.0	14.0	15.3	13.9	15.2	15.5	15.3
% damage	4.0	3.0	3.8	3.8	4.0	5.0	4.0	3.5	2.0
% foreign material	2.9	1.8	4.0	1.0	2.8	6.4	3.0	2.8	2.9
Discounted price/ legal bushel	1.04	1.04	1.01	1.04	1.04	.97	1.04	1.04	1.04
Input price	1.040	1.059	.938	1.049	1.049	.926	1.012	1.003	1.003
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	339	340	401	403	404	405	406	407	408
Test weight	54.5	53.0	56.0	55.0	55.0	54.0	53.0	54.0	55.0
% moisture	13.5	15.3	11.5	15.5	15.0	15.5	15.5	15.5	15.5
% damage	3.0	2.0	83.0	5.0	2.0	4.0	4.9	2.5	3.0
% foreign material	3.0	1.7	3.7	2.6	2.3	2.6	2.9	2.9	2.8
Discounted price/ legal bushel	1.04	1.03	.64	1.04	1.04	1.04	1.03	1.04	1.04
Input price	1.012	.974	.640	1.021	1.021	1.003	.974	1.003	1.021
Bin capacity (bushels)	18,000	18,000	9,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	411	416	417	418	423	424	425	426	427
Test weight	55.4	55.0	54.7	54.5	54.5	53.0	55.0	56.0	54.0
% moisture	12.4	14.3	15.5	15.5	13.3	14.3	14.5	14.3	14.0
% damage	88.0	2.5	3.0	2.8	2.5	7.9	3.0	2.3	3.3
% foreign material	4.9	2.5	2.7	2.4	2.9	9.6	2.8	2.9	5.9
Discounted price/ legal bushel	.605	1.04	1.04	1.04	1.04	.895	1.04	1.04	1.04
Input price	.598	1.021	1.016	1.012	1.012	.847	1.021	1.040	1.003
Bin capacity (bushels)	9,000	18,000	18,000	18,000	9,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	428	429	430	431	432	433	425	436	437
Test weight	55.5	54.0	55.5	56.0	55.0	55.0	56.5	54.5	54.0
% moisture	15.0	15.5	15.5	14.0	13.7	13.7	13.7	15.3	14.5
% damage	4.0	3.3	4.5	1.8	4.3	1.2	2.9	4.0	2.0
% foreign material	2.9	3.0	2.9	2.5	4.0	4.4	1.8	3.0	3.0
Discounted price/ legal bushel	1.04	1.04	1.04	1.04	1.03	1.02	1.04	1.04	1.04
Input price	1.030	1.003	1.030	1.040	1.012	1.002	1.049	1.012	1.003
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	9,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	438	439	440	441	442	443	501	502	503
Test weight	56.0	54.0	56.5	56.5	55.5	56.5	54.5	55.0	55.0
% moisture	13.0	15.2	10.2	13.7	14.0	13.7	15.5	15.0	15.3
% damage	4.5	1.0	.5	4.5	2.5	4.5	2.9	3.0	3.0
% foreign material	2.8	1.5	.8	2.5	3.0	2.5	2.8	2.9	2.8
Discounted price/ legal bushel	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04
Input price	1.040	1.003	1.049	1.049	1.030	1.049	1.012	1.021	1.021
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	504	505	506	507	508	509	510	511	512
Test weight	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
% moisture	15.0	15.5	15.5	14.5	15.0	15.5	15.5	15.0	15.0
% damage	2.9	3.0	4.0	4.0	2.9	3.0	3.0	4.0	2.9
% foreign material	2.7	2.6	1.3	4.7	1.0	2.7	2.6	2.5	2.3
Discounted price/ legal bushel	1.04	1.04	1.04	1.02	1.04	1.04	1.04	1.04	1.04
Input price	1.021	1.021	1.021	1.002	1.021	1.021	1.021	1.021	1.021
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	513	514	516	517	522	523	524	525	526
Test weight	55.0	54.5	55.0	54.0	55.5	55.5	53.0	56.5	55.0
% moisture	14.0	15.5	15.5	15.3	15.5	15.5	15.5	14.2	14.3
% damage	3.0	3.0	3.0	2.4	2.5	2.5	3.0	2.0	2.6
% foreign material	5.9	2.4	2.4	2.8	2.8	2.8	2.8	1.5	1.9
Discounted price/ legal bushel	1.00	1.04	1.04	1.04	1.04	1.04	1.03	1.04	1.04
Input price	.982	1.012	1.021	1.003	1.03	1.03	.974	1.049	1.021
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	527	528	530	531	532	533	534	536	537
Test weight	54.0	54.0	54.5	57.0	55.0	54.5	57.5	57.0	57.0
% moisture	15.2	14.5	14.8	11.7	14.0	15.0	10.3	15.0	12.3
% damage	2.8	1.0	2.0	5.0	3.0	2.2	9.0	2.7	6.0
% foreign material	3.0	2.8	2.8	.5	3.0	1.4	2.0	.7	1.0
Discounted price/ legal bushel	1.04	1.04	1.04	1.04	1.04	1.04	1.02	1.04	1.035
Input price	1.002	1.002	1.011	1.059	1.021	1.012	1.047	1.058	1.054
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	538	539	540	602	604	605	606	607	609
Test weight	56.5	55.0	56.5	54.0	55.0	54.0	55.0	55.0	54.5
% moisture	13.8	14.3	13.9	15.0	15.0	15.5	15.0	15.5	15.5
% damage	4.0	3.0	4.5	2.3	4.0	4.0	3.0	2.9	3.5
% foreign material	2.0	2.9	1.7	2.5	2.0	2.6	2.8	2.4	2.9
Discounted price/ legal bushel	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04
Input price	1.049	1.021	1.049	1.003	1.021	1.003	1.021	1.021	1.012
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000	18,000

Table 13 (Continued)

Bin number	613	615	616	617	618	619	621	623	625
Test weight	55.6	54.0	54.5	53.5	54.5	54.0	54.5	56.0	54.5
% moisture	15.5	15.2	15.3	16.0	15.5	15.4	13.7	12.0	14.3
% damage	4.0	4.0	2.8	3.0	2.7	2.7	2.7	9.0	2.9
% foreign material	2.3	2.9	1.3	2.8	2.5	2.8	2.8	5.0	2.9
Discounted price/ legal bushel	1.04	1.04	1.04	1.02	1.04	1.04	1.04	1.00	1.04
Input price	1.033	1.003	1.012	.974	1.012	1.003	1.012	1.000	1.012
Bin capacity (bushels)	18,000	18,000	18,000	18,000	18,000	18,000	18,000	9,000	18,000

Table 13 (Continued)

Bin number	627	629	643	644	Tank X	Tank Y	Tank Z
Test weight	54.0	54.5	54.5	53.0	56.5	57.0	57.6
% moisture	15.0	15.0	14.3	13.5	13.5	13.9	13.7
% damage	2.9	1.9	3.3	8.9	6.5	4.7	4.4
% foreign material	2.9	2.0	2.8	3.0	1.9	1.7	2.0
Discounted price/ legal bushel	1.04	1.04	1.04	1.005	1.03	1.04	1.04
Input price	1.003	1.012	1.012	.951	1.039	1.059	1.029
Bin capacity (bushels)	18,000	18,000	18,000	9,000	401,000	560,430	585,000